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URBAN HEAT ISLAND: AN ENVIRONMENTAL ECONOMIC MODELING

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ABSTRACT

This paper presents an environmental economic modeling, in which I propose *heat island integral* to represent the force of heat island and *urban heat island function*. Also presented is a *heat island tax/subsidy scheme*; the subsidy based in the numerical data of tree carbon fixation is given to contributers to cooling the city by providing vegetation in the city. In our model any metropolitan resident consumes goods and have landscape gardeners plant trees in his/her gardens, and producers supply goods. Every emitter emits heat and gases in the atmosphere, which results in urban heat island and an anormous amount of cost, so that it is a recent economic problem of great importance for large cities such as Tokyo. This paper tries to give an economic theoretical solution to this prolbem.

1. INTRODUCTION

The Tokyo Metropolis is now observed to have two "suns"; one is the sun in the sky, the other is beneath the surface. The latter may rival solar heat, and Tokyo is now a gigantic *Heat Island* with peaks such as Otemachi (the central business district of Tokyo) and Shinjuku. It has been reported that the temperature for these areas at 18:00 in July 31, 2031 is respectively anticipated as 45.5 [See Saitoh, Ito, Yamada(2006)]. Nevertheless, the Imperial Palace, the Meiji Shrine, and the Shinjuku Gyoen National Garden as *Cool Islands* are cooler, and the temperature of these areas remains rather low compared to the hotter spots mentioned above. Formal definitions of heat island and cool island are given in the next section.

Recently, megacities have been struck by localized torrential downpours of more than 100mm per hour in the summertime, e.g., an area in the Nerima Ward in Tokyo experienced 33.5°C and 131mm/h in July 21, 1999. There were ascending currents and a huge cumulonimbus which was ten times larger than a usual one. Nonetheless, only a few millimeters of raindrops were observed in Musashino City and West Tokyo City, which are not so distant from the Nerima Ward. This is a very distinctive phenomenon of urban heat island. It is considered that breezes from the Tokyo Bay, the Sagami Bay and the Sea of Kashima converged to the area in the Nerima Ward, which made the atmosphere unstable and resulted in localized heavy rains. They were harder than the deluge of rain in the Tokai District in 2000 where 589mm was recorded, and the maximum precipitation per hour was 114mm. With the help of sandbags and water bags, everyone should prepare against the inundation caused by unforeseen urban localized torrential downpours originally due to urban heat island. Tokyo had 15 incidents of heavy rain in the last two decades. The rainfall was 57.5mm/h in October 13, 2003, which was the second highest record in the history of meteorological observation. The power failure due to telephone polls broken by a downpour tangled traffic on the highway and the railroad.

This paper proposes to consider urban atmosphere as a complex of gaseous attributes or characteristics, and the goods or commodities combined by nongaseous attributes that are produced by the firms and the green areas as a composite of biological attributes supplied by *landscape gardeners*. The paper proposes the model where each resident has his/her land planted by a gardener and simultaneously emits heat and trace gases. For this, I adopt an analytical framework of the New Consumer Theory by Gorman and Lancaster Theory. Section 2 presents the periurban model of urban warming in the framework *à la* Gorman-Lancaster and introduces the concept of *Heat Island Integral*. Our model involves residents, offices, manufacturers and farmhouses such as producers, and gardeners who play a very important role to cool down a metropolis by roof-top and wall gardening and tree-planting activities, etc.

2. URBAN HEAT ISLAND IN A PERIURBAN METROPOLIS

2.1 A Model of Urban Heat Island in a Periurban Metropolis

The Characteristics/Functionings model is introduced in this subsection. In the Gorman-Lancasterian New Consumer Theory, goods are regarded as a composition of characteristics. For the sake of simplicity, the two terms, "attributes" and "characteristics", are used interchangeably throughout this paper. All trace gases such as Greenhouse gases(GHGs) can be interpreted as *gaseous attributes* in our framework, since they partially compose the urban atmosphere as an urban public good. This paper focuses upon emitted heat as a intangible attribute, since it is the main cause of urban warming.

It was Luke Howard, an English meteorologist, who first noticed that the urban temperature was higher than the rural one by making observations of climate data in London in 1807~1816. He already calculated that it was 3.7°C higher at nights in the urban area by comparing the observed values of the inside and the outside of London.

Consider a monocentric periurban metropolis which is supposed to be a "climatically closed city," i.e., there are no climatic influences from and on the neighboring cities. "Periurban" means the peripheral zone of the urban area, where reside inhabitants, manufacturers and landscape gardeners. For the sake of simplicity, assume that our periurban metropolis is composed of two regions, i.e., the urban area and the periurban area, which are divided into many blocks, $\beta \in \boldsymbol{\beta} = \{1, \ldots, B\}$: the set of blocks. In other words, our periurban metropolis is represented by a mesh of blocks, such as a grid of the center of Chicago. Assume that whatever size and shape can be chosen for a block, as a smaller size in Tokyo or a larger one in Chicago.

Let there be N residents indexed by $i \in \mathbf{N} = \{1, \ldots, N\}$: the set of inhabitants who live in the periurban belt. Each individual emits heat and trace gases when consuming goods or services indexed by j and its set is $\mathbf{J} = \{1, \ldots, J\}$. For the sake of simplicity, it is assumed that each producer j supplies only one good j which is composed of C characteristics indexed by $c \in \mathbf{C} = \{1, \ldots, C\}$: the set of attributes. Denote as q_{jc} an amount of attribute c embodied in one unit of good j. There are three categories of producers: i.e., officies, manufacturers and farmhouses.

Suppose that resident i chooses a gardener named ℓ to have a part of his/her land planted with trees, and $\boldsymbol{\Lambda} = \{1, \ldots, \Lambda\}$ is the set of gardeners. Define $q_{\ell s}$ as a biomass of species s in one square meter supplied by gardener ℓ, and $\mathbf{S} = \{1, \ldots, S\}$ is the set of species of flora and fauna as *biological attributes*. Let x_{ij} be resident i's consumption of good j, and $A_{i\ell}$ be his/her demand for the green area supplied by gardener ℓ, then, $x_i = (x_{i1}, \ldots, x_{iJ}, A_{i\ell})$ is his/her consumption vector. There is also the metropolitan government whose task is to reduce heat emission by making effective use of a *heat island tax/subsidy scheme* defined below. Not to mention, every inhabitant, producer or gardener resides in some block, so that an index β will be omitted hereafter in almost all the cases, except for describing some variables related to block β.

The urban atmosphere is regarded as a complex of gaseous attributes including trace gases, which are to be mainly generated by production and consumption activities. It is naturally assumed that the amount of gases such as N_2 and O_2 are stationary, so we can focus upon heat and trace gases as attributes in our paper. An index q_{jg} is also used hereafter to identify the gth gaseous attribute, and q_{jH} is an amount emitted of heat as an attribute when producing one unit of good j. Let $\mathbf{G} = \{C+1, \ldots, C+G, H\}$ be the set of intangible attributes such as trace gases and heat in the urban atmosphere.

When producing one unit of good, each producer does not choose but to jointly emit heat and trace gases as vexing by-products, $q_{jH} \geq 0$ and $q_{jg} \geq 0, \forall g \in \mathbf{G}$, which are producer j's unit emission of heat and each gas. Thus, $q_{jH}x_j$ (resp. $q_{jg}x_j$) is firm j's amount emitted of heat (resp. gas) when it produces x_j units of good j. Landscape gardeners also emit heat and gases, $q_{\ell H} \geq 0$ and $q_{\ell g} \geq 0, \forall g \in \mathbf{G}$, which are gardener ℓ's unit emission of gas and heat. Hence, $q_{\ell H}A_\ell$(resp. $q_{\ell g}A_\ell$) is gardener ℓ's emitted quantity of a heat(resp. gas) in his/her production of A_ℓ units of greening service ℓ. Inhabitants also emit heat and gases, $q_{iH} \geq 0$ and $q_{ig} \geq 0, \forall g \in \mathbf{G}$, which are city dweller i's unit emission of gas and heat. Hence, $q_{iH}A_{i\ell}$ and $q_{ig}A_{i\ell}$ are individual i's emitted quantity of a heat and gas in his/her consumption of $A_{i\ell}$ units of greening service ℓ.

Let z_{i0} be resident i's numéraire characteristic that he/she possesses, by which other attributes can be utilized. Amounts of each attribute embodied in the goods and the atmosphere which are consumed by resident i are given by

$$z_{ic} = \sum_{j \in \mathbf{J}} q_{jc} x_{ij}, \ \forall c \in \mathbf{C} \tag{1}$$

and

$$\begin{aligned} z_g = & \sum_{j \in \mathbf{J}} q_{jg} \sum_{i \in \mathbf{N}} x_{ij} + \sum_{j \in \mathbf{J}} q_{jg} A_j + \sum_{\ell \in \boldsymbol{\Lambda}} q_{\ell g} A_{i\ell} \\ & + \sum_{\ell \in \boldsymbol{\Lambda}} \sum_{i \in \mathbf{N}} q_{ig} \sum_{j \in \mathbf{J}} x_{ij} + \sum_{\ell \in \boldsymbol{\Lambda}} \sum_{i \in \mathbf{N}_\ell} q_{ig} A_{i\ell} \ , \ \forall g \in \mathbf{G}. \end{aligned} \tag{2}$$

In the above equation, z_{ic} means the consumption of nongaseous attributes which compose goods, while z_g (resp. z_H) represents the total amount of a trace gas (resp. heat) emitted by all residents, producers and gardeners. Note that the value of $z_g, \forall g \in \mathbf{G}$, can be measured via ton or kilojoule, as in the Introduction. Heat and gases are generated both in the consumption and production of goods. Every inhabitant is made to consume not only his/her emission but also the quantity emitted by the rest of the metropolis. When he/she uses goods, he/she emits heat and gases, which were already released when the goods were made by producers. Both of the above equations may be interpreted as *characteristic availability functions*, which convert commodities into attributes. The amount of any characteristic in each good can be regarded as a parameter that is objective and common to all consumers, i.e., it has a public-good property. Thus, the inhabitants as consumers must behave as "quality takers", since they can only change their consumption of z_{ic} and z_{ig}, via the choice of x_{ij}. Producers and gardeners can choose the composition of attributes embedded in the goods and the services.

A part, $\eta_g z_g$, $0 < \eta_g < 1$, of an aggregate emission of gases is observed to stay in the urban atmosphere and the rest, $(1 - \eta_g) z_g$, is perceived to disintegrate. Of this amount, about 43% of CO_2 emission is absorbed by the oceans and forests as carbon sinks. An integration rate or an inverse of a lifetime of each trace gas is denoted as λ_g, with $0 < \lambda_g < \eta_g, \forall g \in \mathbf{G}$, so that the mass of the gth gas that stays in the urban air is $(\eta_g - \lambda_g) z_g, \forall g \in \mathbf{G}$. Allowing θ_g be a conversion parameter from mass(ton/year) to concentration(ppmv), and $z_g(\gamma)$ be an amount emitted at time γ of the gth trace gas, then an amount of trace gas accumulated from time t_0 to time t, which is converted into a concentration is given by

$$\zeta_g^t = \int_{t_0}^{t} \theta_g^t (\eta_g^t - \lambda_g^t) z_g^t(\gamma) \, d\gamma, \ \forall g \in \mathbf{G} \backslash \{H\}. \tag{3}$$

This can be written as

$$\zeta_g^t = \int_{t_0}^{t} \Theta_g^t z_g^t(\gamma)\, d\gamma,\ \forall g \in \mathbf{G}\backslash\{H\} \tag{4}$$

where $\Theta_g^t \equiv \theta_g^t(\eta_g^t - \lambda_g^t)$ is a climatical parameter related to the gth trace gas. An argument of time t is omitted hereafter. Another equation is proposed for heat as a flow in the next subsection.

It is the local climate in the block, which most influences any economic agent who resides or works in β. However, climatical incidents depend upon not only the concentrations in each block, but also those in the entire metropolis, as exampled in the Nerima incident in the Introduction. Therefore one observes

$$Z = (\zeta_1, \ldots, \zeta_G) \tag{5}$$

which affects all residents, producers and gardeners in the periurban metropolis.

2.2 Heat Island Integral

The next issue is how to represent the heat in our metropolis. In effect, there must be differences in the temperature of building surface, back alleys, rooftops, streets, and green tracts of land, which are directly exposed to the solar radiation. However, these differences of the surface temperature of the ground coverage can be measured by utilizing infrared cameras or remote sensing.

Let A_β be an area of block β and S_β be its level surface projection, i.e., the area which could absorb the solar radiation. More precisely, it is the sum of developable areas of the ground coverage, e.g., the streets, the tree crowns, the rooftops and walls of the buildings, which exist in A_β. Denote $\tau_\beta(u,w)$ as a function of the surface temperature of S_β and $\alpha_\beta(u,w)$ as a function of atmospheric temperature in A_β, where u and w are the plane coordinates. Then, the Riemann sum led me to propose a concept of *Heat Island Integral* which reads

$$\begin{aligned}\Upsilon_\beta &= \frac{1}{S_\beta}\left\{\iint_{S_\beta} \tau_\beta(u,w)\,du\,dw \right.\\ &\left. - \iint_{A_\beta} \alpha_\beta(u,w)\,du\,dw\right\},\ \forall \beta \in \boldsymbol{\beta}. \end{aligned} \tag{6}$$

Needless to say, the existence condition of this multiple integral is that the functions, $\tau_\beta(u,w)$ and $\alpha_\beta(u,w)$, are continuous and compact in the domains S_β and A_β, and it is easily seen that this condition is satisfied.

Υ_β is not the result of heat emission on the part of the economic agents, but it stems from the ground coverage of the metropolis. Υ_β measures the force of urban heat island in each block β, since the temperature of asphalt and concrete is very often higher than that of the atmosphere. It is observed that asphalt and concrete absorb about 90% of the solar radiation, so that the walls of buildings made of concrete absorb approximately the same amount of heat, which result in urban heat island. Let me give some numerical examples of the values in Υ_β. The mercury stood at 27.5 degrees at an observation point in Shinjuku at 14:00 on June 6, 2002, where a wall of a building made of concrete was observed to have the heat of 38°C and a street of asphalt had 45°C. The central area of Shinjuku ($A_\beta = 4\text{km}^2$) must be extended to its plane of projection ($S_\beta = 9.3\text{km}^2$), which is exposed to the solar radiation. Also, heat is emitted from the underground shopping center in Shinjuku. By remote sensing, $\tau_\beta(\cdot)$ and $\alpha_\beta(\cdot)$ can be constructed, thus, one can calculate Υ_β for each block.

The following formula is physically supposed: *Sensible Heat Flux*(ε_β) = *Heat Island Integral*$(\Upsilon_\beta)\times$ *Convective Heat Conductivity*$(\kappa), \forall \beta \in \boldsymbol{\beta}$, where κ varies according to the temperature. Block β is called a *Heat Island if* $\Upsilon_\beta > 0, \forall \beta \in \boldsymbol{\beta}$, and a *Cool Island* if $\Upsilon_\beta \leq 0, \forall \beta \in \boldsymbol{\beta}$. As was mentioned in the Introduction, the Imperial Palace, the Meiji Shrine, and the Shinjuku Gyoen National Garden as cool islands are cooler than the center of Shinjuku, i.e., the heat capacity of the latter was observed to be about 300W/m^2, whereas it was less than 200W/m^2 in the Shinjuku Gyoen National Garden by the measurement of the Ministry of Environment. Furthermore, by its observation, the Arakawa Rivers in Tokyo was much cooler than the above areas, because they nearly absorb all of the solar radiation.

2.3 Vegetation, Water, Heat, and the Urban Warming Function

Let L_i, L_j and L_ℓ be the land owned by resident i, producer j, and gardener ℓ. Also let $A_i \equiv \rho_i L_i$ and $A_j \equiv \rho_j L_j$ be greening areas of their land required to be planted with trees, where ρ_i and ρ_j are greening rates for city dweller i and firm j, which are legally determined, e.g., at least 20% of the rooftop on the houses and buildings, when they are newly built, enlarged or reconstructed in Tokyo. Suppose that gardeners and farmhouses are exempted from the obligation of greening a part of their own lands in our model.

As I assumed that inhabitant i chooses one gardener ℓ to green a part of his/her land A_i with plants, at a cost of σ_ℓ/m^2. Then, each person is to have his/her green area planted with plants, insects and microorganisms in the soil. These living things are regarded as *biological attributes* that may be offered by gardener ℓ as by-products, and let $A_{i\ell}$ be his/her area A_i planted by gardener ℓ. Denote $\mathbf{S}$ as the set of species. City dweller i therefore consumes biological attributes by possessing the green area in his/her land as represented by

$$z_{is} = q_{\ell s} A_{i\ell},\ \forall s \in \mathbf{S},\ \forall \ell \in \mathbf{\Lambda}. \tag{7}$$

Resident i can enjoy seeing creatures in his/her green area: they may be trees, flowers, insects, birds, or minute animals.

Suppose that each producer has an environmental branch which is in charge of planting trees in a part of its lot size A_j, with its cost σ_j. Let $A_P \equiv \sum_{\beta\in\boldsymbol{\beta}} A_\beta$ be the area to be planted with street trees and tree lawns undertaken by public works, and $\Gamma(A_p)$ is its cost. Summing these green areas yields

$$A = \sum_{i\in\mathbf{N}}\sum_{\ell\in\mathbf{\Lambda}} A_{i\ell} + \sum_{j\in\mathbf{J}} A_j + \sum_{\beta\in\boldsymbol{\beta}} A_\beta. \tag{8}$$

Incidentally, passers-by can see hedges of others' houses, and can have beautiful views of parks. Children can amuse themselves in small parks which would stud the cities to and fro.

The quantity of water is crucially important to cool supercities, so underdrained rivers must be resuscitated.

Let W be the total amount of water in the metropolis represented by

$$W = W_0 + W_P \tag{9}$$

where W_0 is the quantity of water that already exists and W_p is the amount which is to be revived by public enterprises with a cost $\Omega(W_p)$, e.g., resuscitating culverts.

By physics, it is easily concluded that the more abundant the amounts of A and W, the cooler the metropolis, i.e., the more, the better. Notice that A and W have a public-good property, since their cooling effects can prevail all the blocks of the metropolis as beneficial externalities. This paper focuses on greening as one of the important natural factors which plays a significant role in cooling down the supercity.

Sensible heat flux from the ground coverage of the metropolis therefore is represented by

$$\sum_{\beta \in \boldsymbol{\beta}} \varepsilon_\beta(A, W, Z) = \kappa \sum_{\beta \in \boldsymbol{\beta}} \Upsilon_\beta \left(A, W, Z\right). \tag{10}$$

Note that this function depends on the planted areas and the amount of water. Both of them can diminish the value of Υ_β by cooling down the ground coverage of the city. However, the above equation is also dependent upon the vector of concentration of trace gases.

Let ε_i, ε_j and ε_ℓ be heat emitted by resident i, firm j, and gardener ℓ in the metropolis. Total heat emission E which affects all economic agents in the metropolis is thus represented as

$$E(A, W, Z) = \sum_{i \in \mathbf{N}} \varepsilon_i + \sum_{j \in \mathbf{J}} \varepsilon_j + \sum_{\ell \in \boldsymbol{\Lambda}} \varepsilon_\ell + \sum_{\beta \in \boldsymbol{\beta}} \varepsilon_\beta \tag{11}$$

The first three terms represent artificial emissions of heat and the last term is the sensible heat flux generated from the ground coverage, all of which are assumed here to depend upon the amounts of vegetation, water, and the vector of concentration of gases.

2.4 Urban Warming Function

Together with the concentrations of GHGs as stocks, total heat emission defines the *Urban Warming Function*:

$$U = U\left(E, Z\right). \tag{12}$$

Urban heat island is a typical example of a public good which is both nonrival and nonexcludable. However, its impact on each resident varies from region to region, which can be treated as a *regional public good.* As was mentioned in the Introduction, urban heat island with 33.5°C occured in some blocks of the Nerima Ward, Tokyo on July 21, 1999, which was not necessarily due to the consumptions and productions of goods in that area, but due to those of districts in Tokyo, as well as the climate conditions of the block at that time. Consequently, the worst that could happen to any block where some climatical conditions are satisfied at some point in time, as in the above unforeseen incident. Let $D_\beta(U)$ is a monetary damage of block β due to urban heat island. With the differentiability of $D_\beta(U)$, it is naturally assumed that $(\partial D_\beta/\partial U)(\partial U/\partial E) > 0$ and $(\partial D_\beta/\partial U)(\partial U/\partial Z) > 0, \forall \beta \in \boldsymbol{\beta}$.

The urban heat island function U depends upon the total heat emission E and the vector of concentrations Z, both of which compose the urban atmosphere of the atmosphere. Urban heat island may now be regarded as a *negative externality* which has been directly provided by the economic agents who emit heat and GHGs into the urban air. Its climate damages can be measured in physical units, which are, in order: i) an increase in heat strokes due to temperature rises, ii) an increase in the occurance of showers and localized torrential downpours, iii) spread of infectious diseases carried, for example, by mosquitoes, iv) changes of biodiversity in an urban ecosystem, etc.

3. DESIGNING A HEAT ISLAND TAX / SUBSIDY SCHEME

3.1 A Heat Island Tax

Let $\mathbf{E} = \mathbf{N} \cup \mathbf{J} \cup \boldsymbol{\Lambda}$ be the set of emitters in the metropolis. For example, consider a driver who emits both heat by using a car with an air-conditioner and gases such as CO_2. Let ε_{e0} be an initial amount of heat emitted by emitter e and ε_e be his/her current emission. Assume that the amount of heat emitted can be converted into CO_2, so that it is possible to construct a cost function, with which each metropolitan emitter e is confronted.

$$C_e = \alpha_e(\varepsilon_{e0} - \varepsilon_e) + t\varepsilon_e, \forall e \in \mathbf{E} \tag{13}$$

where t is a heat island tax rate which may change annually. Then, $t\varepsilon_e$ is an amount of tax payed by emitter e. The objective for each emitter is to minimize the cost which is related to urban heat island.

In order to obtain our desired result, I need another differentiability assumption.

Assumption. For any $e \in \mathbf{E}$, C_e is strictly quasi-concave and twice continuously differentiable with $C_e(0) = 0$.

Simple calculation of cost minimization reads

$$t = \frac{d\alpha_e}{d\varepsilon_e},\ \forall e \in \mathbf{E} \tag{14}$$

which means that the tax rate equals to the marginal abatement cost(MAT) of any emitter. It is concluded therefore that every MAT is same, irrespective of the curvature of cost function of any emitter. Then the sum of tax is

$$T = \sum_{e \in \mathbf{E}} t\varepsilon_e = \sum_{e \in \mathbf{E}} \frac{d\alpha_e}{da_e} \varepsilon_e \tag{15}$$

which may cover the damages due to urban heat island and the rest can be refunded as subsidies for the collaboraters to cool down the city. Thus, one observes

$$T = \sum_{\beta \in \boldsymbol{\beta}} D_\beta(U) + S. \tag{16}$$

3.2 Heat Island Subsidies Based on the Numerical Data of Tree Carbon Fixation

Let the set of trees be $\mathbf{T}$, and let $z_{\tau g}$ be an amount of CO_2 anticipated to be fixed by tree τ. Denote d_τ as tree τ's diameter at the height of a chest and ρ_τ as a coefficient related to tree τ. Denote also ϱ_τ as a coefficient of tree τ and υ_τ be the decision coefficient, then we have in general that

$$z_{\tau g} = \varrho_\tau d_\tau^{\varrho_\tau},\ R^2 = \upsilon_\tau, \forall \tau \in \mathbf{T}. \tag{17}$$

For example, one has $z_{\tau g} = 0.005\varrho^{2.496}$, $R^2 = 0.957$, for ginkgo, in Fujiwara(2004). According to him, Toshima

Ward in Tokyo has a room for vegetation for about 180 thousand of trees. If they will be planted in five years, it is anticipated that 988t-CO_2 will be fixed at the end of the last year.

The CO_2 fixation by any tree reads:

$$z_{\tau g} h_\tau n_\tau = \varrho_\tau d_\tau^{\varrho_\tau} h_\tau n_\tau,\ R^2 = \upsilon_\tau, \forall \tau \in \mathbf{T}. \tag{18}$$

Let $n_{i\ell\tau}$ be the number of trees that resident i commissions landscape gardener ℓ to plant his/her garden. And $n_{j\tau}$ is assumed to be the number that producer j plants in its site of firm or factory. Let also $n_{\beta\tau}$ be the number of trees planted by some public work in block β. If the amount of vegetation is denoted $V_\tau = h_\tau d_\tau n_\tau, \forall \tau \in \mathbf{T}$, then the CO_2 fixation by all trees is given by the following formula:

$$\begin{aligned} &\sum_{\tau \in \mathbf{T}} z_{\tau g} h_\tau n_\tau \\ =&\sum_{\tau \in \mathbf{T}} z_{\tau g} h_\tau \left(\sum_{\ell \in \boldsymbol{\Lambda}} \sum_{i \in \mathbf{N}} n_{i\ell\tau} + \sum_{j \in \mathbf{J}} n_{j\tau} + \sum_{\beta \in \boldsymbol{\beta}} n_{\beta\tau} \right). \end{aligned} \tag{19}$$

As greening lands and rooftops are one of the most effective ways to cool the metropolis, let the metropolitan government determine $0 < \varphi_i < 1$ as the refund rate for greening the area $A_{i\ell}$ that resident i requests landscape gardener ℓ. Assume that the value of φ_i is decided by the scientific data about tree-planting. As defined, σ_ℓ is gardener ℓ's greening cost per square meter, hence, $\sigma_{i\ell} A_{i\ell}$ is resident i's total cost, and $\varphi_i \sigma_{i\ell} A_{i\ell}$ is ℓ's refund for the effort to plant trees. Candidates of subsidy sharing are determined by tree-planting density, i.e.

$$\varphi_i = \varphi_i \left(\frac{\sum_{\tau \in \mathbf{T}} z_{\tau g} h_\tau n_\tau}{A_{i\ell}} \right), \forall i \in \mathbf{N}. \tag{20}$$

Similarly, for producers one observes

$$\varphi_j = \varphi_j \left(\frac{\sum_{\tau \in \mathbf{T}} z_{\tau g} h_\tau n_\tau}{A_j} \right), \forall j \in \mathbf{J} \tag{21}$$

Heat island subsidies are therefore as follows:

$$S = \sum_{i \in \mathbf{N}} \sum_{\ell \in \boldsymbol{\Lambda}} \varphi_i \sigma_{i\ell} A_{i\ell} + \sum_{j \in \mathbf{J}} \varphi_j \sigma_j A_j. \tag{22}$$

In the above equation, the first term is the subsidy given to residents and the second term is the subsidy which goes to producers.

This paper has applied the economic way of thinking to an environmental problem of growing importance and has proposed an environmental economic modeling of urban heat island. My discussion has proceeded on the premise that the urban atmosphere is a composite of gaseous characteristics as well as heat as an important attribute. Sato(2006) also incorporates farmhouses and technological innovations to cool urban heat islands.

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